# Before you begin

- These slides are used in presentations at workshops.
- They are best viewed with a pdf reader like Acrobat Reader (free download).
  - Make sure that "Single Page View" or "Fit to Window" is selected.
  - Navigation buttons are provided at the bottom of each screen if needed (see below).
- Viewing in web browsers is not recommended.

#### Do not try to print the slides

There are many more pages than the number of slides listed at the bottom right of each screen!

Apologies for any inconvenience.



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# Inferential Statistics (testing hypotheses) $(m\alpha+hs)Smart$ Workshop Semester 2, 2016

Geoff Coates

(This workshop is a follow-up to the earlier "Introduction to Statistical Inference" session.) These slides go through the steps used to conduct the one sample t-test and demonstrates how to extract the necessary information from a description of an experiment.



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# What can $(m\alpha+hs)Smart$ do for you?

#### Online Stuff

- presentation slides from workshops on many topics
- practice exercises
- short videos
- and more!

#### Drop-in Study Sessions

 Monday, Wednesday, Friday, 10am-12pm, Ground Floor
 Barry J Marshall Library, teaching weeks and study breaks.

#### Workshops

• See our current <u>Workshop Calendar</u> for this <u>Semester's topics</u>.

#### Email: geoff.coates@uwa.edu.au

- Can't find what you want?
- Got a question?
  Drop us a line!



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- ▶ Go Data and test statistic
- ▶ <u>●</u> Decision
- Using a t-table to find the p-value [some stats units only]
- $\bigcirc$  Checking the assumptions for the *t*-test [some stats units only]

Another typical test/exam-style question

- Setting up the hypotheses
- Data and test statistic
- ▶ 💿 p−value
- ▶ 💿 Decision
- $\bullet$  Go Using a *t*-table to find the *p*-value [some stats units only]
- $\sim$  Go Checking the assumptions for the *t*-test [some stats units only]

Appendix: A tip for understanding t-tests  $\bigcirc$   $\bigcirc$ 



Statistical inference is a technique for *inferring* something about an entire population.



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Statistical inference is a technique for *inferring* something about an entire population.





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**Statistical inference** is a technique for *inferring* something about an entire population. The "something" is a numerical characteristic called a population parameter.





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**Statistical inference** is a technique for *inferring* something about an entire population. The "something" is a numerical characteristic called a population parameter.



population parameters:



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# population parameters: population mean: $\mu$



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**Statistical inference** is a technique for *inferring* something about an entire population. The "something" is a numerical characteristic called a population parameter.



#### population parameters:

population mean:  $\mu$  population standard deviation:  $\sigma$ 



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**Statistical inference** is a technique for *inferring* something about an entire population. The "something" is a numerical characteristic called a population parameter.



#### population parameters:

- population mean:  $\boldsymbol{\mu}$
- population standard deviation:  $\sigma$ 
  - population proportion: p



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**Statistical inference** is a technique for *inferring* something about an entire population. The "something" is a numerical characteristic called a population parameter. We do this by collecting a random sample of population members



#### population parameters:

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- population standard deviation:  $\boldsymbol{\sigma}$ 
  - population proportion: p

**Note:** separating population parameters and sample statistics helps organise all the notation used in statistics.



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All hypothesis testing procedures follow the same general structure:



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**Null Hyp. (H**<sub>0</sub>): A specific statement about a population parameter (or parameters).



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  - **Data:** Random sample(s) from the population(s).
  - **Test Statistic:** Suitable estimate of the population parameter (or combination of parameters) derived from these data.
- Sampling Dist<sup>n</sup>: Describes the probability structure for the test statistic when  $H_0$  is true.
  - p-value: probability of observed test statistic value or one more favourable to  $H_1$  from the sampling distribution.

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# A typical test/exam-style question

We will now apply this general framework to a commonly used test, the one-sample t-test, using data from a published paper:

Franklin, D et al 2000, 'Oral Health Status of Children in a Paediatric Intensive Care Unit', *Intensive Care Medicine*, vol. 26, pp. 319-324.



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Franklin, D et al 2000, 'Oral Health Status of Children in a Paediatric Intensive Care Unit', *Intensive Care Medicine*, vol. 26, pp. 319-324.

Some of the analysis in this paper has been re-cast as a typical test/exam-style question.



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A study of the dental status of critically ill children in a Paediatric Intensive Care Unit examined 16 children with permanent teeth and found that the mean number of missing or filled teeth was 1.2 with a standard deviation of 1.9. Extensive analysis has established that the mean number of such teeth in the wider population of children is 1.4. Test whether the mean for critically ill children differs from this.





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 $H_0$  :  $\mu$ 



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 $\mathsf{H}_1: \underline{\mu}$ 



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 $H_1: \mu \neq 1.4$  missing/filled teeth



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 $H_0: \mu = 1.4$  missing/filled teeth  $H_1: \mu \neq 1.4$  missing/filled teeth



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**Data:** n = 16 critically ill children with permanent teeth.



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 $\overline{x} = 1.2$  teeth.



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 $\overline{x} = 1.2$  teeth. In most Intro Stats units the only available testing procedure is the *one-sample* t-*test*. This uses a "stan-dardized" version of  $\overline{x}$ :

$$t = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}} =$$



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$$t = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1.2 - 1.4}{\frac{1.9}{\sqrt{16}}}$$



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$$t = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1.2 - 1.4}{\frac{1.9}{\sqrt{16}}} = -0.421$$



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*p*-value: probability of the observed test statistic value or one more favourable to  $H_1: \mu \neq 1.4$  when  $H_0$  is true.



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We observed  $\overline{x} = 1.2$  (or equivalently  $t = \frac{\overline{x} - 1.4}{0.475} = -0.421$ ):





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We observed  $\overline{x} = 1.2$  (or equivalently  $t = \frac{\overline{x} - 1.4}{0.475} = -0.421$ ):



What sort of values of  $\overline{x}$  would have been more favourable to H<sub>1</sub>?



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We observed  $\overline{x} = 1.2$  (or equivalently  $t = \frac{\overline{x} - 1.4}{0.475} = -0.421$ ):



What sort of values of  $\overline{x}$  would have been more favourable to H<sub>1</sub>? Well, anything further away from 1.4 than the observed 1.2, such as 1.1 or 0.9, etc (ie. any  $\overline{x} < 1.2$ ).



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These values correspond to t < -0.421.



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These values correspond to t < -0.421.

In fact, since  $H_1$  is two sided, any value further away from 1.4 on the other side (ie. greater than 1.6) would have been more favourable to  $H_1$  as well.



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*p*-value: probability of the observed test statistic value or one more favourable to  $H_1: \mu \neq 1.4$  when  $H_0$  is true.

We observed  $\overline{x} = 1.2$  (or equivalently  $t = \frac{\overline{x} - 1.4}{0.475} = -0.421$ ):



What sort of values of  $\overline{x}$  would have been more favourable to H<sub>1</sub>? Well, anything further away from 1.4 than the observed 1.2, such as 1.1 or 0.9, etc (ie. any  $\overline{x} < 1.2$ ).

#### These values correspond to t < -0.421.

In fact, since  $H_1$  is two sided, any value further away from 1.4 on the other side (ie. greater than 1.6) would have been more favourable to  $H_1$  as well.

These values correspond to t > 0.421 (a useful feature of t).

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p-value: probability of the observed test statistic value or one more favourable to  $H_1:~\mu\neq 1.4$  when  $H_0$  is true.





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*p*-value: probability of the observed test statistic value or one more favourable to  $H_1$ :  $\mu \neq 1.4$  when  $H_0$  is true.



Now, the question is "what is the probability of getting a t value in the red zone when H<sub>0</sub> is true?"



Next

*p*-value: probability of the observed test statistic value or one more favourable to  $H_1$ :  $\mu \neq 1.4$  when  $H_0$  is true.



Now, the question is "what is the probability of getting a t value in the red zone when  $H_0$  is true?" To answer that, we need to know the **Sampling Distribution** of t. Statistical theory says that the distribution looks like this ...



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Now, the question is "what is the probability of getting a *t* value in the red zone when  $H_0$  is true?" To answer that, we need to know the **Sampling Distribution** of *t*. Statistical theory says that the distribution looks like this ... a "*t*-distribution with 16 - 1 = 15 degrees of freedom (df)". So the *p*-value is this shaded area.



Image: Image

A typical test/exam-style question: p-value



Before we calculate the p-value, have a guess at what you think it is. (Hint: the total area under the curve is 1).



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A typical test/exam-style question: *p*-value



Before we calculate the p-value, have a guess at what you think it is. (Hint: the total area under the curve is 1).

Technology such as Excel can give an exact answer: the cell function "=T.DIST.2T(0.421,15)" returns p - value = 0.680.

Inferential Statistics (testing hypotheses)

A typical test/exam-style question: p-value



Before we calculate the p-value, have a guess at what you think it is. (Hint: the total area under the curve is 1).

Technology such as Excel can give an exact answer: the cell function "=T.DIST.2T(0.421,15)" returns p - value = 0.680. Was your guess close?

Inferential Statistics (testing hypotheses)

A typical test/exam-style question: decision

p - value = 0.680

**Decision:** if p-value "too small" (ie. < significance level  $\alpha$ ), we reject  $H_0$  in favour of  $H_1$  at the  $100\alpha\%$  significance level.



Inferential Statistics (testing hypotheses)

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A typical test/exam-style question: decision

p - value = 0.680

Clearly a p-value this big does *not* meet the criterion of "too small" and we would "retain H<sub>0</sub> at any sensible significance level".



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A typical test/exam-style question: decision

p - value = 0.680

Clearly a p-value this big does *not* meet the criterion of "too small" and we would "retain H<sub>0</sub> at any sensible significance level".

Some Intro Stats units use a *t*-table to find a suitable approximation for the *p*-value. If you don't use this method, you can skip the next section by clicking <a href="https://www.new.approximation.com">https://www.new.approximation.com</a>



Next

• A *t*-table provides enough information about a *p*-value to make decisions:





◆ Prev )

• A *t*-table provides enough information about a *p*-value to make decisions:

						Tail pro	bability	p				
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	5.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	5.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.851	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.185	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	5.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.850	1.058	1.310	1.708	2.060	2.107	2.480	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.102	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.475	2.771	3.057	3.421	3.690
28	0.685	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.765	3.047	3.408	3.074
29	0.083	0.854	1.055	1.311	1.699	2.045	2.150	2.402	2.750	3.038	2.390	3.039
30	0.085	0.854	1.055	1.310	1.097	2.042	2.147	2.407	2.750	3.030	3.305	3.040
40	0.081	0.851	1.050	1.303	1.084	2.021	2.123	2.423	2.704	2.971	3.307	3.331
50	0.679	0.849	1.047	1.299	1.671	2.009	2.109	2.403	2.078	2.937	3.201	3.490
80	0.679	0.848	1.045	1.290	1.671	2.000	2.099	2.390	2.000	2.715	3.232	2.416
100	0.678	0.846	1.043	1.292	1.660	1.990	2.088	2.374	2.039	2.00/	2.174	2,200
1000	0.677	0.845	1.042	1.290	1.000	1.984	2.081	2.304	2.020	2.8/1	3.174	3.390
2*	0.674	0.842	1.037	1.282	1.645	1.962	2.050	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%





Contents

• A *t*-table provides enough information about a *p*-value to make decisions:

						Tail pro	bability	( p				
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.6
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.9
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.61
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3,365	4.032	4.773	5.893	6.86
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5,95
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3,499	4.029	4.785	5.40
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.04
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.78
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.58
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.43
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.31
13	0.694	0.870	1.079	1.350	1 771	2 160	2 282	2.650	3.012	3 372	3.852	4.22
14	0.692	0.868	1.076	1 345	1.761	2 145	2 264	2 624	2 077	3 326	3 787	4 14
15	0.691	0.866	1.074	1 3.41	1 753	2 131	2 240	2.602	2 047	3 286	3 733	4.07
16	0.690	0.865	1.071	1 3 3 7	1 746	2 1 2 0	2 235	2 583	2 0 2 1	3 252	3.686	4.01
17	0.690	0.863	1.069	1 3 3 3	1.740	2 110	2 224	2 567	2 808	3 222	3 646	3.06
1.9	0.689	0.862	1.067	1 220	1 724	2 101	2 214	2 552	2.878	3 1 9 7	3 611	3.07
10	0.689	0.861	1.066	1 229	1 720	2.002	2 205	2 5 2 0	2 861	3 174	3 570	3.88
20	0.687	0.860	1.064	1 225	1 725	2.095	2 107	2.529	2.845	2 1 5 2	3 552	2.85
21	0.687	0.850	1.062	1 222	1 721	2.080	2.197	2.519	2.045	2 1 2 5	2 5 2 7	2.81
22	0.000	0.039	1.065	1 221	1 717	2.000	2.109	2.510	2.001	2 1 1 0	3 505	2 70
22	0.000	0.030	1.060	1.321	1.714	2.014	2.103	2.500	2.019	2 104	2 495	2.76
2.5	0.005	0.030	1.050	1.319	1.711	2.009	2.177	2.300	2.007	2.001	2.467	2.74
24	0.085	0.037	1.059	1.316	1.702	2.004	2.172	3 492	2.797	2.079	2.450	2.72
20	0.004	0.856	1.058	1.310	1.706	2.000	2.167	2.400	2.707	2.047	2.425	2.70
20	0.004	0.850	1.058	1.313	1.700	2.050	2.102	2.472	2.779	2.057	2,421	2.60
20	0.004	0.855	1.057	1.314	1.705	2.052	2.150	2.4/3	2.771	3.037	2.400	3.09
20	0.085	0.855	1.050	1.313	1.701	2.048	2.154	2.407	2.703	3.047	2,206	3.07
29	0.083	0.854	1.055	1.311	1.699	2.045	2.150	2.402	2.750	3.038	2.390	3.03
30	0.085	0.854	1.055	1.310	1.097	2.042	2.147	2.457	2.750	3.030	3.305	3.04
40	0.681	0.851	1.050	1.303	1.684	2.021	2.125	2.425	2.704	2.971	3.307	3.55
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	5.261	3.49
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	5.2.32	3.40
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	5.41
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	5.174	3.39
000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.30
Ζ*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.29
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.99

• Each row refers to a different t-distribution. In this case we need the row for 15 df.



Inferential Statistics (testing hypotheses)

• A t-table provides enough information about a p-value to make decisions:

						Tail pro	bability	p				
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
15	0.691	0.866	1.074	1.341	1 753	2 1 3 1	2 2 4 9	2 602	2 947	3.286	3.733	4.073
16						2.120	2.235	2.583	2.921	3.252		
Ζ."	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2,576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

• Each row refers to a different t-distribution. In this case we need the row for 15 df.



Inferential Statistics (testing hypotheses)

• A *t*-table provides enough information about a *p*-value to make decisions:

H     .23       1     1.00       2     0.83       3     0.76       4     0.77       5     0.77       6     0.77       7     0.71       8     0.77       11     0.65       12     0.66       13     0.66       14     0.65       15     0.66       16     0.65       17     0.68       18     0.68       20     0.61       21     0.65       22     0.63       23     0.64       24     0.64       25     0.64       26     0.64       27     0.64       28     0.64       30     0.64       400     0.65       50     0.67						Tail pro	bability	( p					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005	
2 0.81 3 0.74 4 0.77 5 0.77 7 0.77 10 0.77 11 0.66 12 0.66 13 0.67 10 0.77 11 0.66 12 0.66 13 0.66 14 0.66 15 0.66 16 0.68 17 0.77 10 0.77 10 0.77 11 0.66 12 0.66 12 0.66 12 0.66 12 0.66 12 0.66 13 0.66 14 0.66 15 0.66 15 0.66 16 0.66 16 0.66 17 0.77 10 0.66 12 0.66	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6	
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7 0.71 7 0.71 9 0.77 9 0.77 1 0.65 4 0.77 1 0.65 4 0.65 5 0.65 6 0.65 6 0.65 6 0.65 8 0.68 8 0.68 9 0.68													/
8     0.77       9     0.77       11     0.66       13     0.69       13     0.69       13     0.69       14     0.66       15     0.66       16     0.68       17     0.68       0.68     0.68       0.06     0.68       0.06     0.68       0.06     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68       0.68     0.68 <td></td> <td>/</td>													/
9 9 0.77 0 0.77 0 0.77 1 0.68 1 0.66 1 0.													
00     0.761       11     0.693       22     0.653       0.633     0.664       14     0.654       0.693     0.664       14     0.654       0.693     0.664       0.693     0.664       0.615     0.664       0.612     0.664       0.612     0.664       0.612     0.664       0.612     0.664       0.55     0.664       0.55     0.664       0.612     0.664       0.612     0.664       0.612     0.664       0.612     0.664       0.613     0.664       0.614     0.664       0.614     0.614       0.614     0.614       0.614     0.614       0.614     0.614       0.614     0.614       0.614     0.614       0.614     0.614       0.614     0.614       0.614     0.614       0.614     0.													/
111     0.66       112     0.66       113     0.65       115     0.66       115     0.66       115     0.66       116     0.66       117     0.68       118     0.68       119     0.68       119     0.68       120     0.68       121     0.68       123     0.68       124     0.68       125     0.68       126     0.68       127     0.68       128     0.68       129     0.68       129     0.68       129     0.68       120     0.68       120     0.68       120     0.68       120     0.68       120     0.68       120     0.68       120     0.68       120     0.68       120     0.68       120     0.68       120     0.68 <td></td> <td>a little in the little in the second second</td>													a little in the little in the second second
12     0.65       13     0.65       14     0.65       15     0.66       16     0.65       17     0.68       18     0.65       20     0.65       21     0.66       22     0.65       23     0.68       24     0.68       25     0.68       26     0.68       27     0.68       28     0.68       29     0.63       29     0.63       20     0.65       20     0.65       20     0.65       20     0.65       20     0.65       20     0.65       21     0.63       25     0.63       26     0.63       27     0.63       28     0.63       29     0.63       20     0.65       20     0.65													
13     0.65       14     0.65       15     0.66       16     0.65       17     0.68       18     0.65       19     0.68       20     0.68       21     0.65       22     0.68       23     0.63       24     0.63       25     0.68       26     0.63       27     0.63       28     0.63       29     0.63       30     0.61       40     0.63       50     0.65													
14     0.65       15     0.65       16     0.65       17     0.61       18     0.62       19     0.63       21     0.65       22     0.63       23     0.63       24     0.63       25     0.63       26     0.63       27     0.63       28     0.63       29     0.63       30     0.63       40     0.63       50     0.65													
115     0.65       116     0.85       117     0.65       118     0.68       119     0.65       20     0.65       21     0.65       22     0.65       23     0.66       24     0.65       25     0.65       26     0.65       27     0.65       28     0.66       29     0.63       30     0.65       50     0.67       50     0.67													
6     0.69       77     0.68       9     0.686       90     0.686       11     0.681       12     0.681       13     0.681       14     0.681       15     0.681       16     0.681       17     0.681       18     0.661       19     0.681       10     0.681	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073 <	values
7     0.68       8     0.68       9     0.68       1     0.68       2     0.68       3     0.68       5     0.68       6     0.68       6     0.68       0     0.68												4.015	
8     0.68       9     0.68       0.0     0.68       1.1     0.68       2.2     0.68       3.3     0.68       4.4     0.68       5.5     0.68       6.6     0.68       7     0.68       9     0.68       9     0.68       0     0.68       0     0.68       0     0.68       0     0.68       0     0.68       0     0.68													
9     0.68       10     0.68       11     0.68       12     0.68       13     0.68       14     0.68       15     0.68       16     0.68       17     0.68       18     0.68       19     0.68       10     0.68       10     0.68       10     0.68       10     0.68       10     0.68       10     0.68       10     0.68													
20     0.68       21     0.68       22     0.68       13     0.68       14     0.68       15     0.68       16     0.68       17     0.68       18     0.68       19     0.68       10     0.68       10     0.68       10     0.68													
21 0.68 22 0.68 23 0.68 24 0.68 25 0.68 26 0.68 27 0.68 27 0.68 29 0.68 30 0.68 30 0.68 40 0.68 50 0.67													
22 0.65 23 0.65 24 0.65 25 0.65 26 0.65 27 0.65 27 0.65 29 0.65 30 0.65 30 0.65 40 0.65													
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c 0.0.											2012/21	1204021	

• Each row refers to a different t-distribution. In this case we need the row for 15 df.



Inferential Statistics (testing hypotheses)

• A *t*-table provides enough information about a *p*-value to make decisions:

						Tail pro	bability	( p					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005 🧲	— right tail probab
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6	
													$\frown$
													/ \
													/ \
												4.781	
													0
													_
												4.140	
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073	values of t
τ."	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291	
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%	

• Each row refers to a different t-distribution. In this case we need the row for 15 df.



Inferential Statistics (testing hypotheses)

TABLE D	t dist	ribution	critical v	alues									
	Tail probability p												
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005 🗲	right tail probability
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6	
													$\frown$
													/ \
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In this case, the positive version of t is 0.421



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In this case, the positive version of t is 0.421 and the smallest value of t in this row is 0.691.

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In this case, the positive version of t is 0.421 and the smallest value of t in this row is 0.691. We can now say that the blue shaded area is 0.25.

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In this case, the positive version of t is 0.421 and the smallest value of t in this row is 0.691. We can now say that the blue shaded area is 0.25.

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So, we can say that the p-value (grey area partially obscured by the blue area) is greater than the blue area:



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So, we can say that the p-value (grey area partially obscured by the blue area) is greater than the blue area:

$$p - \text{value} > 2 \times 0.25$$
  
 $p - \text{value} > 0.5.$ 



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So, we can say that the p-value (grey area partially obscured by the blue area) is greater than the blue area:

$$p-value > 2 \times 0.25$$

p - value > 0.5.

(Remember that the exact answer is p - value = 0.680).



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This is enough to make our decision:



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This is enough to make our decision:

Clearly, since p - value > 0.5 does not meet the criterion of "too small", we would "retain H<sub>0</sub> at any sensible significance level" as before.

#### Assumptions for the t-test

 You may also be asked to consider whether the assumptions required for a hypothesis test to work have been met. If you don't discuss this topic in your unit, you can skip the next section by clicking



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Most testing procedures come with assumptions. These are conditions that the population(s) and sample(s) have to meet for the p-values to be reliable.



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For *all* tests, a key assumption is

the data is a random sample of the relevent population.



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The population here is "all critically ill children with permanent teeth in Paediatric Intensive Care".

We have to assume that the chosen 16 children represent a random sample of such children  $\ldots$ 



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For the one sample t-test we just performed, a key assumption is

the population follows a Normal Distribution or the sample size is large.



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 $\overline{x} = 1.2$  teeth s = 1.9 teeth



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Still can't see it? If not, here is another clue:

the smallest possible data value here is 0 missing/filled teeth.

You can't go backwards from the mean by even one standard deviation! This means that there must be data values much bigger than 1.2 in order to create such a large standard deviation.



Here's a possible data set that fits these summary figures:





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Here's a possible data set that fits these summary figures:



Technically, the t-test should not have been used here.



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Here's a possible data set that fits these summary figures:



Technically, the t-test should not have been used here. Alternative methods do exist when these assumptions are not met. (If your unit covers "non-parametric tests" or "data transformations" you may know some of them.)



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### Another t-test question: setting up the hypotheses

Plaque develops on teeth in response to the presence of bacteria and can lead to harmful effects. The difference in plaque coverage (% of all teeth surfaces with plaque) between admission and discharge was measured for each of the 16 critically ill children in the previous example. The mean of these differences (discharge – admission) was 4.0% with a standard deviation of 7.4%. Test whether there was a mean change in plaque coverage between admission and discharge.





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$$t = \frac{\overline{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}} =$$



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*p*-value: probability of the observed test statistic value or one more favourable to  $H_1$ :  $\mu_d \neq 0$  when  $H_0$  is true.



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What sort of values of  $\overline{x}$  would have been more favourable to  $H_1$ ?



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*p*-value: probability of the observed test statistic value or one more favourable to  $H_1$ :  $\mu_d \neq 0$  when  $H_0$  is true.

We observed  $\overline{x} = 4.0$  (or equivalently  $t = \frac{\overline{x}_d - 0}{1.85} = 2.162$ ):



What sort of values of  $\overline{x}$  would have been more favourable to H<sub>1</sub>? Well, anything further away from 0 than the observed 4.0, such as 4.5 or 5.0, etc (ie. any  $\overline{x} > 4.0$ ).



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In fact, since  $H_1$  is *two* sided, any value further away from 0 on the other side (ie. less than -4.0) would have been more favourable to  $H_1$  as well.



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These values correspond to t < -2.162 (a useful feature of t).

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Since the sample size is still 16, the **Sampling Distribution** of t is still t(15)



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Since the sample size is still 16, the **Sampling Distribution** of t is still t(15) and the p-value is this shaded area.



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Before we calculate the p-value, have a guess at what you think it is. (Hint: the total area under the curve is 1).



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Before we calculate the p-value, have a guess at what you think it is. (Hint: the total area under the curve is 1).

• Using Excel again to get the exact answer: "=T.DIST.2T(2.162,15)" returns p - value = 0.047.



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p - value = 0.047



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p - value = 0.047

If  $\alpha$  is chosen to be 0.05 (5%), our *p*-value does meet the criterion of "too small" (just) and we would "reject H<sub>0</sub> at the 5% significance level".



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p - value = 0.047

**Decision:** if p-value "too small" (ie. < significance level  $\alpha$ ), we reject H<sub>0</sub> in favour of H<sub>1</sub> at the 100 $\alpha$ % significance level.

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In fact, it seems that plaque tends to *increase* because the difference was calculated as discharge % – admission%!



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Another t-test question: decision

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Another t-test question: decision

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• If your unit doesn't cover t-tables, you can skip the next section by clicking  $\bigcirc$  here

Next

						Tail pro	obability	p					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005 <	right tail probabilit
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6	-
													$\wedge$
													Decamber
													0
												4.140	volues of t
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073	values of t
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Let's take a close up look at the grey shaded area.



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Let's take a close up look at the grey shaded area.



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Let's take a close up look at the grey shaded area.

From the *t*-table, our *t* = 2.162 sits between 2.131 WESTERN WISTRALIA STUDYSmarter Inferential Statistics (testing hypotheses) Contents (Prev) Next



Let's take a close up look at the grey shaded area.

From the *t*-table, our t = 2.162 sits between 2.131 and 2.249. WESTERN WISTRALIA WITH WALL AND ALL AND



Let's take a close up look at the grey shaded area.

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So, we don't know the exact p-value but we can say that the p-value (grey area) is



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So, we don't know the exact p-value but we can say that the p-value (grey area) is larger than the blue area (0.02)



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So, we don't know the exact p-value but we can say that the p-value (grey area) is larger than the blue area (0.02) but smaller than the green area (0.025).



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So, we don't know the exact p-value but we can say that the p-value (grey area) is larger than the blue area (0.02) but smaller than the green area (0.025). Remembering to double the above values to add in the left-hand tail, we can say:

 $2 \times 0.02$ 

$$0.04$$



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0.04

(Recall that the exact answer is p - value = 0.047.)

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 $0.04 < p-\mathsf{value} < 0.05$ 



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Image: A Prev (

0.04

This is enough to make our decision:



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0.04

This is enough to make our decision:

If  $\alpha$  is chosen to be 0.05 (5%), our *p*-value does meet the criterion of "too small" (just) and we would "reject H<sub>0</sub> at the 5% significance level" as before.



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### Assumptions for the t-test

 You may also be asked to consider whether the assumptions required for a hypothesis test to work have been met. If you don't discuss this topic in your unit, you can skip the next section by clicking



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For the one sample t-test we just performed, a key assumption is

the population follows a Normal Distribution or the sample size is large.



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However, outliers are still a potential problem (in this case a child who acquires or loses an unusually large amount of plaque while in hospital). You would need the raw data to check.



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There should be something familiar about the t-distribution.



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There should be something familiar about the t-distribution. It looks a lot like a Standard Normal distribution (shown above in grey).



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This means that a *t*-statistic is like a *z*-score for the sample mean (when  $\mu = 1.4$ ). That is, it roughly follows the "68-95-97.5% Rule":



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So, t = -0.421 (ie. half a standard deviation below the population mean) is a pretty typical result when  $\mu = 1.4$ .



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On the other hand, t = 2.162 is (roughly) 2 standard deviations above the mean so it's an unusual result when  $\mu = 1.4$ . In other words, we probably do have compelling evidence against H<sub>0</sub>.



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#### Using STUDY Smarter Resources

This resource was developed for UWA students by the STUDY*Smarter* team for the numeracy program. When using our resources, please retain them in their original form with both the STUDY*Smarter* heading and the UWA crest.



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